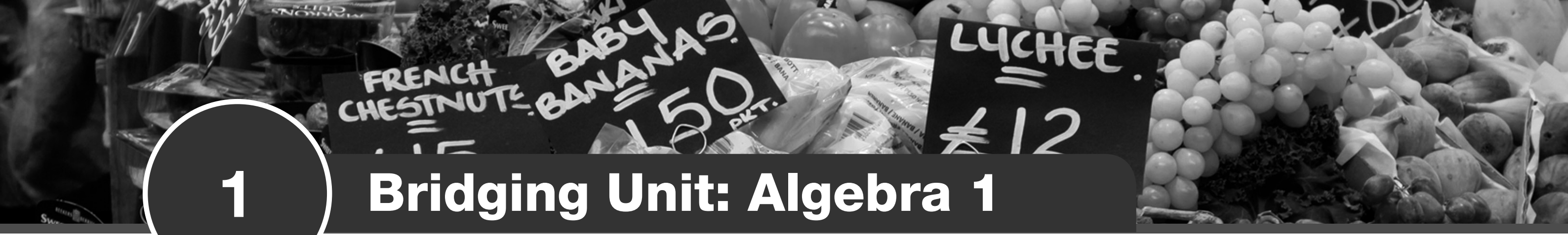

1 Bridging Unit: Algebra 1

Topic D: Completing the square

Bridging to Ch1.4

Some quadratics are **perfect squares** such as $x^2 - 8x + 16$ which can be written $(x-4)^2$. For other quadratics you can **complete the square**. This means write the quadratic in the form $(x+q)^2 + r$

Key point

The completed square form of $x^2 + bx + c$ is $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

If you have an expression of the form $ax^2 + bx + c$ then first factor out the a, as shown in Example 1

Example 1

Write each of these quadratics in the form $p(x+q)^2 + r$ where *p*, *q* and *r* are constants to be found.

a $x^2 + 6x + 7$ **b** $-2x^2 + 12x$

a $x^2 + 6x + 7 = \left(x+\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 7$

$= (x+3)^2 - 9 + 7$

$= (x+3)^2 - 2$

The constant term in the bracket will be half of the coefficient of *x*

b $-2x^2 + 12x = -2\left[x^2 - 6x\right]$

$= -2\left[(x-3)^2 - 9\right]$

$= -2(x-3)^2 + 18$

First factor out the coefficient of x^2 then complete the square for the expression in the square brackets.

Try It 1

Write each of these quadratics in the form $p(x+q)^2 + r$

a $x^2 + 22x$ **b** $2x^2 - 8x - 6$ **c** $-x^2 + 10x$

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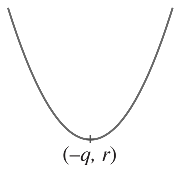
The turning point on the curve with equation $y = p(x+q)^2 + r$ has coordinates $(-q, r)$, this will be a minimum if p is positive and a maximum if p is negative.

Key point



Example 2

Find the coordinates of the turning point of the curve with equation $y = -x^2 + 5x - 2$

$$-x^2 + 5x - 2 = -\left[x^2 - 5x + 2\right]$$

$$= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 2\right]$$

$$= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{17}{4}\right]$$

$$= -\left(x - \frac{5}{2}\right)^2 + \frac{17}{4}$$

So the maximum point is at $\left(\frac{5}{2}, \frac{17}{4}\right)$

First factor out the –1 then complete the square for the expression in the square brackets.

The curve is at its highest point when the bracket is equal to zero: $x - \frac{5}{2} = 0 \Rightarrow x = \frac{5}{2}$

Try It 2

Find the coordinates of the turning point of each of these curves and state whether they are a maximum or a minimum.

a $y = x^2 - 3x + 1$ **b** $y = -x^2 - 7x - 12$ **c** $y = 2x^2 + 4x - 1$

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Bridging Exercise Topic D

1 Write each of these quadratic expressions in the form $p(x+q)^2+r$

a x^2+8x

b x^2-18x

c x^2+6x+3

d $x^2+12x-5$

e $x^2-7x+10$

f x^2+5x+9

g $2x^2+8x+4$

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h $3x^2+18x-6$

i $2x^2-10x+3$

j $-x^2+12x-1$

k $-x^2+9x-3$

l $-2x^2+5x-1$

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2 Use completing the square to find the turning point of each of these curves and state whether it is a maximum or a minimum.

a $y = x^2 + 14x$

b $y = x^2 - 18x + 3$

c $y = x^2 - 9x$

d $y = -x^2 + 4x$

e $y = x^2 + 11x + 30$

f $y=-x^2+6x-7$

g $y=2x^2+16x-5$

h $y=-3x^2+15x-2$

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